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On a possibility to construct gauge invariant quantum formulation for non-gauge classical theory

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Abstract

A non-gauge dynamical system depending on parameters is considered. It is shown that these parameters can have such values that corresponding canonically quantized theory will be gauge invariant. The equations allowing to find these values of parameters are derived. The prescription under consideration is applied to obtaining the equation of motion for tachyon background field in closed bosonic string theory.

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1 Introduction

Procedure of canonical quantization is a natural and correct base for construction of consistent quantum theory and it is applied as a ground for any specific quantization methods. Formulation of new fundamental models of quantum field theory leads, as a rule, to necessity of studying new aspects of canonical quantization. At present the most general approach to canonical quantization of arbitrary systems is BFV-method [1]-[9] (see also [10, 11]) which includes all the previous achievements of other quantization methods.

In this paper we would like to discuss a new interesting aspect of canonical quantization arising from the string theory in background fields [12]-[17]. In this theory one considers a string interacting with massless background fields and introduces the Weyl invariance principle according to which the renormalized trace of energy-momentum tensor should vanish. General structure of the trace was studied in refs. [18, 19].

The consideration carried out in refs. [12]-[19] in schematic form looks as follows. Let us consider for simplicity the bosonic string theory only. In this case set of dynamical variables consists of string coordinates X^{μ} and components of two-dimensional world sheet metric $g_{ab}(\tau, \sigma)$. Classical lagrangian includes the Fradkin-Tseytlin term [13, 14] describing string interaction with dilaton field. Presence of this term spoils Weyl invariance of the classical theory and trace of classical energy-momentum tensor does not vanish identically. Nevertheless, effective action depending on external fields g_{ab} is considered and condition of its independence on conformal factor of two-dimensional metric is imposed. This condition leads to equations of motion for background fields and their explicit form can be found perturbatively.

It is important that there is no Weyl anomaly and its cancellation in such a theory. The initial classical theory is not Weyl invariant and therefore there is no sense to say about anomalies. An analogous situation also arises in string theory interacting with tachyon field or with massive background field of higher levels [21, 22] where classical theory is not Weyl invariant but at the quantum level Weyl invariance takes place under equations of motion for background fields. Note that this approach suggests calculation of functional integral in a strictly specified way. Namely, one should first calculate the integral over string coordinates and then demand effective action to be Weyl invariant. Thereafter the dependence on two-dimensional metric will be reduced to dependence on a finite number of parameters defining the world

sheet topology.[2].

From general point of view the above situation can be formulated as follows. Consider a non-gauge classical theory action which depends on some parameters, in string theory their role being played by background fields. If the given parameters are constrained by special restrictions then the effective action satisfies identities defining quantum gauge invariant theory. The problem is how to describe such a situation in terms of canonical quantization.

It is well known that any quantum gauge theory is characterized by the operators of first class constraints and their algebra is generated by the nilpotency condition of canonical BRST-charge Ω [1]-[9]. The constraints operators are usually constructed on the base of classical theory constraints. However, if classical theory is not gauge-invariant it has no first class constraints at all and the operator Ω can not be constructed. Thus, from the point of view of standard canonical quantization the situation when gauge quantum theory corresponds to non-gauge classic theory looks very strange. At the same time, we believe that any adequate quantization procedure should be in agreement with canonical quantization[3].

This paper is devoted to a possible way of solving this problem. We introduce some prescription allowing to construct the operator Ω starting from non-gauge classical theory which depends on some parameters and show that this operator Ω will be nilpotent only under special equations for the parameters. Then we derive equation of motion for tachyon background field in closed bosonic string theory using the introduced recipe and demonstrate its efficiency.

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[2]In the recent paper [23] an attempt to treat the integral over string coordinates and world sheet metric without taking into account any special order of integration was undertaken.

[3]In particular, for bosonic string interacting with background gravitational field this agreement was established [27]. However, in that case we had no the problem under consideration.

2 General Formulation

Let us consider a quantum system with the Hamiltonian of the following form:

$$H = H_0(a) + \lambda^\alpha T_\alpha(a). \tag{1}$$

Here λ^α, $a \equiv a_I$ are classical parameters of the theory, $H_0(a) \equiv H_0(q^i, p_i|a)$, $T_\alpha(a) \equiv T_\alpha(q^i, p_i|a)$; (q^i, p_i) are operators of canonically conjugated dynamical variables. Since λ^α are given parameters we cannot consider $T_\alpha(a)$ as constraints.

We suggest that operators $T_\alpha(a)$ can be presented in the form

$$T_\alpha(a) = T_\alpha^{(0)}(a) + T_\alpha^{(1)}(a), \tag{2}$$

where

$$\begin{aligned} [T_\alpha^{(0)}(a), T_\beta^{(0)}(a)] &= i\hbar T_\gamma^{(0)}(a) U_{\alpha\beta}^\gamma(a), \\ [H_{(0)}(a), T_\alpha^{(0)}(a)] &= i\hbar T_\gamma^{(0)}(a) V_\alpha^\gamma(a). \end{aligned} \tag{3}$$

However

$$\begin{aligned} [T_\alpha(a), T_\beta(a)] &= i\hbar T_\gamma(a) U_{\alpha\beta}^\gamma(a) + i\hbar A_{\alpha\beta}(a) \\ [H_{(0)}(a), T_\alpha(a)] &= i\hbar T_\gamma(a) V_\alpha^\gamma(a) + i\hbar A_\alpha(a) \end{aligned} \tag{4}$$

and the operators $A_{\alpha\beta}(a)$, $A_\alpha(a)$ do not vanish in the classical limit. It means that corresponding classical theory is not gauge invariant.

Let us introduce operators Ω and H formally following the BFV-method as if T_α corresponded to first class constraints:

$$\begin{aligned} \Omega &= c^\alpha T_\alpha(a) - \frac{1}{2} U_{\alpha\beta}^\gamma(a) O(\mathcal{P}_\gamma c^\alpha c^\beta) \\ H &= H_{(0)}(a) + V_\alpha^\gamma(a) O(\mathcal{P}_\gamma c^\alpha) \end{aligned} \tag{5}$$

where O denotes some suitable ordering of ghost operators $\mathcal{P}_\gamma$, c^α.

It is not difficult to show that

$$\begin{aligned} \Omega^2 &= \frac{1}{2}([T_\alpha(a), T_\beta(a)] - i\hbar T_\gamma(a) U_{\alpha\beta}^\gamma(a) + i\hbar G_{\alpha\beta}(a)) c^\alpha c^\beta \\ \frac{d\Omega}{dt} &= c^\alpha \frac{\partial T_\alpha(a)}{\partial t} - \frac{1}{2} \frac{\partial U_{\alpha\beta}^\gamma(a)}{\partial t} O(\mathcal{P}_\gamma c^\alpha c^\beta) \\ &- (i\hbar)^{-1}([H_{(0)}(a), T_\alpha(a)] - i\hbar T_\gamma(a) V_\alpha^\gamma(a) + i\hbar G_\alpha(a) c^\alpha) \end{aligned} \tag{6}$$

where $G_{\alpha\beta}(a)$ and $G_\alpha(a)$ are possible ghost contributions. Here we took into account the possible explicit dependence of the operators $T_\alpha(a)$ on time[4]. The eqs.(4,6) lead to

$$\begin{aligned}\Omega^2 &= \frac{1}{2} i\hbar E_{\alpha\beta}(a) c^\alpha c^\beta \\ \frac{d\Omega}{dt} &= \left(\frac{\partial T_\alpha(a)}{\partial t} - E_\alpha(a)\right) c^\alpha - \frac{1}{2}\frac{\partial U^\gamma_{\alpha\beta}(a)}{\partial t} O(\mathcal{P}_\gamma c^\alpha c^\beta)\end{aligned} \tag{7}$$

where

$$\begin{aligned}E_{\alpha\beta}(a) &= A_{\alpha\beta}(a) + G_{\alpha\beta}(a) \\ E_\alpha(a) &= A_\alpha(a) + G_\alpha(a)\end{aligned} \tag{8}$$

We suppose that the operators $E_{\alpha\beta}(a)$ and $\partial T_\alpha(a)/\partial t - E_\alpha(a)$ can be presented as linear combinations of an irreducible set of operators $O_M \equiv O_M(q^i, p_i)$:

$$\begin{aligned}E_{\alpha\beta}(a) &= E^M_{\alpha\beta}(a) O_M \\ \frac{\partial T_\alpha(a)}{\partial t} - E_\alpha(a) &= E^M_\alpha(a) O_M\end{aligned} \tag{9}$$

where $E^M_{\alpha\beta}(a)$, $E^M_\alpha(a)$ are c-valued functions of the parameters a. Then one gets

$$\begin{aligned}\Omega^2 &= \frac{1}{2} ih E^M_{\alpha\beta}(a) O_M c^\alpha c^\beta \\ \frac{d\Omega}{dt} &= E^M_\alpha(a) O_M c^\alpha - \frac{1}{2}\frac{\partial U^\gamma_{\alpha\beta}(a)}{\partial t} O(\mathcal{P}_\gamma c^\alpha c^\beta)\end{aligned} \tag{10}$$

We see that in general case $\Omega^2 \neq 0$, $d\Omega/dt \neq 0$. But the theory can not be called an anomalous one since classical symmetries are absent and it does not make sense to say about their violation at quantum level[5].

Let us suppose that there exists a solution of the following system of equations for the parameters a:

$$\begin{aligned}E^M_{\alpha\beta}(a) &= 0 \\ E^M_\alpha(a) &= 0\end{aligned} \tag{11}$$

[4]Formulation of BFV-procedure for theories explicitly depending on time was given in ref. [32].

[5]The general discussion of anomalies in the BFV-approach can be found in ref. [28].

We also suppose that the equation

$$\frac{\partial U^{\gamma}_{\alpha\beta}(a)}{\partial t} = 0$$

is fulfilled. Let us denote corresponding solution for the parameters as $a^{(0)} \equiv a^{(0)}_I$. Then

$$\begin{aligned} \Omega^2 \Big|_{a\,=\,a^{(0)}} &= 0 \\ \frac{d\Omega}{dt}\Big|_{a\,=\,a^{(0)}} &= 0 \end{aligned} \tag{12}$$

Thus, when parameters a in eqs.(1,2,4,5) take the values $a^{(0)}$ the equations $\Omega^2 = 0$, $d\Omega/dt = 0$ defining a quantum gauge theory should take place. As a result we get the prescription allowing to construct a gauge invariant quantum formulation for non-gauge classical theory depending on parameters[6].

3 Application to String Coupled to Tachyon Background Field

To illustrate efficiency of the above recipe we consider a derivation of linear equation of motion for the tachyon field in closed bosonic string theory.[7] The theory is described by the following action:

$$S = -(2\pi\alpha')^{-1} \int d^2\sigma \sqrt{-g}\{g_{ab}\partial_a X^{\mu}\partial_b X^{\nu}\eta_{\mu\nu} + Q(X)\} \tag{13}$$

Here $\sigma^a \equiv (\tau, \sigma)$; $a, b = 0, 1$; $\mu, \nu = 0, 1, \ldots, D-1$; $\eta_{\mu\nu}$ is Minkowski metric of D-dimensional background space-time, $Q(X)$ is tachyon background field.

[6]In specific theories the equations (11) may have no solutions and corresponding gauge-invariant quantum formulation may not exist at all. Also a part of equations (11) may be fulfilled identically. An example of theory where all the equations (11) are fulfilled identically was given in the paper [22].

[7]It is well known that to obtain the tachyon equation of motion consistent with structure of string amplitudes one should use a nonperturbative consideration (see e.g. [24, 25]). We restrict ourselves to linear approximation since our purpose here is just to illustrate that the above prescription really works.

It is easy to show that

$$g_{ab}\frac{\delta S}{\delta g_{ab}} = -(2\pi\alpha')^{-1}\sqrt{-g}Q(X) \tag{14}$$

Therefore, if the metric g_{ab} was a dynamical variable then the corresponding classical equations of motion would lead to $Q(X) = 0$. The analogous situation takes place for string theory interacting with either the dilaton field [26] or the massive higher spin background fields [21, 22]. To fulfill classical equations of motion for two-dimensional metric one should set the dilaton field to be constant and all the higher massive background fields equal to zero. In order to avoid this situation we have to conclude that components of the metric g_{ab} should be treated as external fields. Such a conclusion is consistent with general ideology accepted in string theory interacting with background fields [12] - [21] where functional integral is calculated only over variables X^μ and metric components g_{ab} are considered as external fields.

Let us parametrize the metric g_{ab} as follows [26]

$$g_{ab} \;=\; e^\gamma \begin{pmatrix} \lambda_1^2 - \lambda_0^2 & \lambda_1 \\ \lambda_1 & 1 \end{pmatrix} \tag{15}$$

It is easy to show that Hamiltonian of the theory (13) has the form

$$H = \int d\sigma \,(\lambda_0 T_0 + \lambda_1 T_1) \tag{16}$$

where

$$\begin{aligned} T_0 &= \frac{1}{2}\Big((2\pi\alpha')P_\mu P^\mu + (2\pi\alpha')^{-1}X'_\mu X'^\mu\Big) + (2\pi\alpha')^{-1}e^\gamma Q(X) \\ T_1 &= P_\mu X'^\mu \end{aligned} \tag{17}$$

Here P_μ are momenta canonically conjugated to coordinates X^μ and $X'^\mu = \partial_\sigma X^\mu$. λ_0, λ_1 are external fields and role of the parameters a is played by the tachyon field $Q(X)$. γ is an external field also. The eq.(16) corresponds to the case $H_0 = 0$.

Let us introduce the functions

$$\begin{aligned} L &= \frac{1}{2}(T_0 - T_1) = \frac{1}{4}(2\pi\alpha')^{-1}\left((2\pi\alpha')P - X'\right)^2 \\ &+ (4\pi\alpha')^{-1}e^\gamma Q(X) \\ \bar{L} &= \frac{1}{2}(T_0 + T_1) = \frac{1}{4}(2\pi\alpha')^{-1}\left((2\pi\alpha')P_+X'\right)^2 \\ &+ (4\pi\alpha')^{-1}e^\gamma Q(X) \end{aligned} \tag{18}$$

It is evident that

$$L = L^{(0)} + L^{(1)}, \bar{L} = \bar{L}^{(0)} + \bar{L}^{(1)} \tag{19}$$

where $L^{(0)}$ and $\bar{L}^{(0)}$ represent standard constraints of the free string theory satisfying Virasoro algebra and

$$L^{(1)} = \bar{L}^{(1)} = (4\pi\alpha')^{-1} e^{\gamma} Q(X) \tag{20}$$

We pay attention that the functions L and $\bar{L}$ are not constraints and they do not satisfy any algebra.[8]

Let us pass now to quantum theory. We choose conformal gauge for the external fields $\lambda_0 = 1$, $\lambda_1 = 0$. In order to obtain linear equation for Q it is sufficient to suppose that X^μ satisfy free string equation of motion $\Box X^\mu = 0$. It means that we can introduce zero and oscillating string modes operators by the standard way.

Let $: L(\tau, \sigma) :$, $: \bar{L}(\tau, \sigma) :$ are the operators corresponding to the classical functions (18) ordered according to the prescription defined in the Appendix A. We introduce the operators

$$\begin{aligned} : L_n(\tau) : &= \int_0^{2\pi} d\sigma \, e^{-\imath n\sigma} : L(\tau, \sigma) : \\ : \bar{L}_n(\tau) : &= \int_0^{2\pi} d\sigma \, e^{\imath n\sigma} : \bar{L}(\tau, \sigma) : \end{aligned} \tag{21}$$

Our main aim is to construct the eqs.(11) in the case under consideration. Let us start with the first of these equations and find the operators corresponding to $A_{\alpha\beta}$ (4) and $G_{\alpha\beta}$ (6). The eqs.(3) show that in order to find $A_{\alpha\beta}$ it is necessary to calculate the commutators $[: L_n :, : L_m :]$, $[: \bar{L}_n :, : \bar{L}_m :]$, $[: L_n :, : \bar{L}_m :]$ where $: L_n :$ and $: \bar{L}_m :$ are given by (21).

To compute the above commutators we introduce symbols L_n and $\bar{L}_m$ of operators $: L_n :$ and $: \bar{L}_m :$ associated with the given ordering prescription.

It is well known that calculation of the commutators can be reduced to calculation of so called $*$-commutators for the corresponding symbols [30, 31]. An example of such a calculation in string theory is given in ref. [26].

[8] If we considered λ_0, λ_1 and γ as dynamical variables then, of course, T_0 and T_1 would be constraints, $Q(X) = 0$ and the corresponding algebra of first class constraints would have standard form of Virasoro algebra. We study another situation when $Q(X) \neq 0$. In this case λ_0, λ_1 and γ are external fields, T_0 and T_1 are not constraints and their Poisson brackets are not expressed in terms of T_0 and T_1.

Let us denote the $*$-commutator of the symbols under consideration as $[L_n, L_m]_*$, $[\bar{L}_n, \bar{L}_m]_*$, $[L_n, \bar{L}_m]_*$. Calculating these $*$-commutators we obtain some integrals (see Appendix B) which unfortunately are ill defined and we should use some regularization procedure. Since we are constructing the calculating scheme within the canonical approach we have no possibility to apply standard regularization procedures accepted for regularization of Feynman integrals in covariant approaches. To regularize the integrals under consideration we have used the method given in Appendix B.

The result of calculations of $*$-commutators can be written as follows[9]

$$
\begin{aligned}
[L_n, L_m]_* &= \hbar(n-m)L_{n+m} + \hbar^2\alpha_0(n-m)\delta_{n,-m} + \frac{D}{12}\hbar^2(n^3-n)\delta_{n,-m} \\
&- (4\pi\alpha')^{-1}\hbar(n-m)\int_0^{2\pi} d\sigma\, e^{-\imath(n+m)\sigma} e^{\gamma(\tau,\sigma)}\,(1+\alpha'\hbar\Box/4)\,Q(X) \\
[\bar{L}_n, \bar{L}_m]_* &= \hbar(n-m)\bar{L}_{n+m} + \hbar^2\beta_0(n-m)\delta_{n,-m} + \frac{D}{12}\hbar^2(n^3-n)\delta_{n,-m} \\
&- (4\pi\alpha')^{-1}\hbar(n-m)\int_0^{2\pi} d\sigma\, e^{\imath(n+m)\sigma} e^{\gamma(\tau,\sigma)}\,(1+\alpha'\hbar\Box/4)\,Q(X) \\
[L_n, \bar{L}_m]_* &= -(4\pi\alpha')^{-1}\hbar(n-m)\int_0^{2\pi} d\sigma\, e^{\imath(m-n)\sigma} e^{\gamma(\tau,\sigma)}\,(1+\alpha'\hbar\Box/4)\,Q(X) \\
&- (4\pi\alpha')^{-1}i\hbar\int_0^{2\pi} d\sigma\, e^{\imath(m-n)\sigma}\left(e^{\gamma(\tau,\sigma)}\right)' Q(X), \qquad (22)
\end{aligned}
$$

where D is dimension of the target space. The eqs.(22) define the form of functions $A_{\alpha\beta}$ (4) in our case. Ghost contribution $G_{\alpha\beta}$ has the standard form as in the free string theory.

As a result the first of eqs.(11) in our theory gives standard conditions $\alpha_0 = \beta_0 = 1$, $D = 26$ and new conditions

$$(\Box + m^2)Q(X) = 0 \qquad (23)$$

$$\gamma'(\tau, \sigma) = 0 \qquad (24)$$

where m^2 is the mass square of tachyon string mode. The equation $\partial U^\gamma_{\alpha\beta}/\partial\tau$ is fulfilled automatically in our case. The second of eqs.(11) leads to the

[9] We pay attention that regularized $*$-commutators do not depend on the regularization parameter. It means that the composite operator Ω^2 is finite automatically and it does not need renormalization in the theory under consideration.

following condition under the eqs.(23, 24)[10]

$$\dot{\gamma}(\tau, \sigma) = 0 \tag{25}$$

The eq.(23) is the known free equation of motion for the tachyon string mode. The eqs.(24, 25) mean that the string world sheet should be flat. Thus the above quantization procedure will be consistent if the world sheet is flat and the field $Q(X)$ in the Lagrangian (13) satisfies the tachyon equation of motion. Note that in this case the operators $:L_n:$ and $:\bar{L}_n:$ form quantum Virasoro algebra.

4 Summary

We have suggested the prescription allowing in certain cases to construct a quantum gauge formulation starting from non-gauge classical theory. The crucial role in this formulation is played by the eqs.(11) defining values which the theory parameters should take to provide consistency of the formally used BFV-procedure.

To illustrate this prescription we have considered the closed bosonic string in tachyon background field. We have found that the above prescription works very well and allows to obtain correct free equation of motion for background field corresponding to tachyon string mode. Quantum Virasoro algebra on the flat world sheet takes place under this equation in spite of absence of any constraints in the initial classical theory.

The obtained results show that the above procedure can be considered as a general method allowing to construct gauge invariant quantum theory for an initially non-gauge classical model. In particular we hope that this method provides a possibility to derive non-linear equations of motion for strings interacting with massless and massive background fields in framework of canonical quantization.

[10]If the equations (23, 24) are fulfilled then $d\Omega/d\tau \sim \dot{\gamma}$ because of special structure of the Hamiltonian (16). By the way we have just the case when the functions T_α depend explicitly on time.

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Appendix A

Let us consider standard mode expansion for the operators X^μ and P^μ (see e.g. [29, 33])

$$\begin{aligned} X^\mu &= \frac{1}{\sqrt{2\pi}} x_0^\mu + \sqrt{2\pi\alpha'} p_0^\mu \tau \\ &+ \frac{i\sqrt{\alpha'}}{\sqrt{2}} \sum_{n\neq 0} \frac{1}{n} (\alpha_n^\mu e^{-\imath n(\tau-\sigma)} + \bar{\alpha}_n^\mu e^{-\imath n(\tau+\sigma)}), \\ P^\mu &= \frac{1}{\sqrt{2\pi\alpha'}} p_0^\mu + \frac{1}{2\pi\sqrt{\alpha'}\sqrt{2}} \sum_{n\neq 0} (\alpha_n^\mu e^{-\imath n(\tau-\sigma)} + \bar{\alpha}_n^\mu e^{-\imath n(\tau+\sigma)}) \end{aligned} \tag{A1}$$

where the operators of zero modes x_0^μ, p_0^μ and of the oscillating ones α_n^μ, $\bar{\alpha}_n^\mu$ satisfy the following commutation relations:

$$[x_0^\mu, p_0^\nu] = i\hbar\delta_\nu^\mu,\ [\alpha_m^\mu, \alpha_n^\nu] = [\bar{\alpha}_m^\mu, \bar{\alpha}_n^\nu] = \hbar m \delta_{m,-n} \eta^{\mu\nu} \tag{A2}$$

We denote an arbitrary ordered operator A depending on x_0^μ, p_0^μ, α_n^μ, $\bar{\alpha}_n^\mu$ as $O(A)$. The most general form of $O(\alpha_n^\mu \alpha_m^\nu)$, $O(\bar{\alpha}_n^\mu \bar{\alpha}_m^\nu)$, $O(x_0^\mu p_0^\nu)$, $O(p_0^\nu x_0^\mu)$ can be written as follows

$$\begin{aligned} O(\alpha_n^\mu \alpha_m^\nu) &= (1-c_{nm})\alpha_n^\mu \alpha_m^\nu + c_{nm} \alpha_m^\nu \alpha_n^\mu, \\ O(\bar{\alpha}_n^\mu \bar{\alpha}_m^\nu) &= (1-\bar{c}_{nm})\bar{\alpha}_n^\mu \bar{\alpha}_m^\nu + \bar{c}_{nm} \bar{\alpha}_m^\nu \bar{\alpha}_n^\mu, \\ O(x_0^\mu p_{\nu 0}) &= (1-c_0) x_0^\mu p_{\nu 0} + c_0 p_{\nu 0} x_0^\mu, \\ O(p_{\nu 0} x_0^\mu) &= (1-\bar{c}_0) p_{\nu 0} x_0^\mu + \bar{c}_0 x_0^\mu p_{\nu 0} \end{aligned} \tag{A3}$$

where the parameters c_{mn}, $\bar{c}_{mn}$, c_0, $\bar{c}_0$ characterize a specific choice of ordering prescription. The commutation relation (A2) and the symmetries of eq.(A3) lead to the properties

$$1 - c_{mn} = c_{nm} \equiv c_n \delta_{n,-m},\ 1 - \bar{c}_{mn} = \bar{c}_{nm} \equiv \bar{c}_n \delta_{n,-m},\ 1 - \bar{c}_0 = c_0 \qquad \text{(A4)}$$

Remember that every specific choice of the quantities c_n, $\bar{c}_n$, and c_0 leads to a strictly defined type of ordering prescription. For example, in the string models the Weyl ordering for zero modes and the Wick ordering for the oscillating ones is usually used (see e.g. [26]). In this case

$$c_n = \bar{c}_n = \Theta(n),\ c_0 = \frac{1}{2} \qquad \text{(A5)}$$

The eqs.(A3) define the most general form of ordering prescription for zero and oscillating string modes. (The ordering prescription given by (A5) will be called the normal ordering and the normal form of an operator A will be denoted as $\mathcal{N}(A)$.)

It is well known [29, 33] that in the case of free string the following relations take place

$$\begin{array}{rcl} L_n^{(0)} & \rightarrow & O(L_n^{(0)}) = \mathcal{N}(L_n^{(0)}) - \hbar\alpha_0\delta_{n,0}, \\ \bar{L}_n^{(0)} & \rightarrow & O(\bar{L}_n^{(0)}) = \mathcal{N}(\bar{L}_n^{(0)}) - \hbar\beta_0\delta_{n,0} \end{array} \qquad \text{(A6)}$$

It is natural to consider only those ordering prescriptions (A3) which are consistent with the eqs.(A6) accepted in the free string theory. It means that the eqs.(A6) can be treated as some restrictions on arbitrary parameters c_{mn}, $\bar{c}_{mn}$ and c_0 in the eqs.(A3,A4). If we demand the eqs.(A6) to take place and the parameters α_0, β_0 to have fixed values we see that c_{mn}, $\bar{c}_{mn}$, c_0 should depend on the parameters α_0 and β_0. Straightforward calculations with the use of the eqs.(A3 - A6) and the definitions of $L^{(0)}$ and $\bar{L}^{(0)}$ lead to

$$\alpha_0 = -\frac{D}{2}\sum_{n>0} n(1 - c_n + c_{-n}),\ \beta_0 = -\frac{D}{2}\sum_{n>0} n(1 - \bar{c}_n + \bar{c}_{-n}) \qquad \text{(A7)}$$

A solution (not general but nevertheless acceptable for our aims) of these equations looks as follows

$$\begin{array}{l} c_0 = 1/2,\ 1 - c_n = c_{-n} = -\alpha_0 \dfrac{(1-\mu)^2}{\mu D}\mu^n \\ 1 - \bar{c}_n = \bar{c}_{-n} = -\beta_0 \dfrac{(1-\mu)^2}{\mu D}\mu^n,\ n > 0,\ |\mu| < 1 \end{array} \qquad \text{(A8)}$$

where μ is an arbitrary parameter. We suppose as usually that $c_0 = \frac{1}{2}$. As a result all the coefficients c_{mn}, $\bar{c}_{mn}$, c_0 and $\bar{c}_0$ are defined now in terms of the given parameters α_0, β_0 and an additional parameter μ. Using the obtained coefficients c_{mn}, $\bar{c}_{mn}$, c_0 and $\bar{c}_0$ we find

$$
\begin{aligned}
:\alpha_n^\mu \alpha_m^\nu: &= (1+\alpha_0 \frac{(1-\mu)^2}{\mu D}\mu^n \delta_{n,-m})\alpha_n^\mu \alpha_m^\nu - \alpha_0 \frac{(1-\mu)^2}{\mu D}\mu^n \delta_{n,-m}\alpha_m^\nu \alpha_n^\mu, \\
:\bar{\alpha}_n^\mu \bar{\alpha}_m^\nu: &= (1+\beta_0 \frac{(1-\mu)^2}{\mu D}\mu^n \delta_{n,-m})\bar{\alpha}_n^\mu \bar{\alpha}_m^\nu - \beta_0 \frac{(1-\mu)^2}{\mu D}\mu^n \delta_{n,-m}\bar{\alpha}_m^\nu \bar{\alpha}_n^\mu, \\
:x_0^\mu p_{\nu 0}: &= \frac{1}{2}(x_0^\mu p_{\nu 0} + p_{\nu 0} x_0^\mu) \qquad \text{(A9)}
\end{aligned}
$$

Here and further we apply the notation $: A :$ for any operator A ordered according to the prescription (A9).

Appendix B

A symbol of an operator is a $c-$valued function of phase variables corresponding to some operator ordering. Let A is the symbol of operator $\hat{A}$, B is the symbol of operator $\hat{B}$. Then the symbol corresponding to operator $\hat{A}\hat{B}$ is denoted $A * B$ and looks like this [30, 31]:

$$
A * B = \exp(\underbracket{\Gamma_1^M \Gamma_2^N} \frac{\delta}{\delta \Gamma_1^M} \frac{\delta}{\delta \Gamma_2^N}) A(\Gamma_1) B(\Gamma_2)\Big|_{\Gamma_1 = \Gamma_2 = \Gamma} \qquad \text{(B1)}
$$

where

$$
\underbracket{\Gamma_1^M \Gamma_2^N} = \Gamma_1^M \Gamma_2^N - :\Gamma_1^M \Gamma_2^N: \qquad \text{(B2)}
$$

are fundamental contractions of the canonical variables Γ^M. The symbol corresponding to commutator of operators has the form

$$
[A, B]_* = A * B - B * A \qquad \text{(B3)}
$$

We apply formalism of operators symbols to calculate quantum algebra of constraints (21). Phase space variables Γ^M in our theory are:

$$
\Gamma^M = (X^\mu(\tau, z), P^\mu(\tau, z)) \qquad \text{(B4)}
$$

where $z \equiv e^{\imath\sigma}$ and we use the ordering prescription given in the Appendix A.

According to the eqs.(B3,B1) $*-$commutator of symbols has the following general structure: $[A, B]_* = i\hbar\{A, B\} + \Delta^{(2)}_{[A,B]} + O(\hbar^3)$ where $\{A, B\}$ is Poisson bracket and $\Delta^{(2)}_{[A,B]}$ is proportional to $\hbar^2$. The straightforward but tedious enough calculations lead to the following explicit form of $\Delta^{(2)}_{[A,,B]}$:

$$
\begin{aligned}
&\Delta^{(2)}_{[A,B]} = \oint \frac{dz_1}{iz_1} \oint \frac{dz_2}{iz_2} \{([\underbrace{X_1^{\mu}(\tau, z_1) P_1^{\nu}}_{}(\tau, z_2)]_A \\
&\times(\underbrace{X_2^{\mu}(\tau, z_1) X_2^{\nu}}_{}(\tau, z_2) \frac{\partial^2 A}{\partial X_1^{\mu} \partial X_2^{\mu}}(\tau, z_1) \frac{\partial^2 B}{\partial P_1^{\nu} \partial X_2^{\nu}}(\tau, z_2) \\
&+\underbrace{X_2'^{\mu}(\tau, z_1) X_2^{\nu}}_{}(\tau, z_2) \frac{\partial^2 A}{\partial X_1^{\mu} \partial X_2'^{\mu}}(\tau, z_1) \frac{\partial^2 B}{\partial P_1^{\nu} \partial X_2^{\nu}}(\tau, z_2) \\
&+\underbrace{X_2^{\mu}(\tau, z_1) X_2'^{\nu}}_{}(\tau, z_2) \frac{\partial^2 A}{\partial X_1^{\mu} \partial X_2^{\mu}}(\tau, z_1) \frac{\partial^2 B}{\partial P_1^{\nu} \partial X_2'^{\nu}}(\tau, z_2) \\
&+\underbrace{P_2^{\mu}(\tau, z_1) P_2^{\nu}}_{}(\tau, z_2) \frac{\partial^2 A}{\partial X_1^{\mu} \partial P_2^{\mu}}(\tau, z_1) \frac{\partial^2 B}{\partial P_1^{\nu} \partial P_2^{\nu}}(\tau, z_2) \\
&+\underbrace{X_2'^{\mu}(\tau, z_1) X_2'^{\nu}}_{}(\tau, z_2) \frac{\partial^2 A}{\partial X_1^{\mu} \partial X_2'^{\mu}}(\tau, z_1) \frac{\partial^2 B}{\partial P_1^{\nu} \partial X_2'^{\nu}}(\tau, z_2)) \\
&+[\underbrace{X_1'^{\mu}(\tau, z_1) P_1^{\nu}}_{}(\tau, z_2)]_A \\
&\times(\underbrace{X_2^{\mu}(\tau, z_1) X_2^{\nu}}_{}(\tau, z_2) \frac{\partial^2 A}{\partial X_1'^{\mu} \partial X_2^{\mu}}(\tau, z_1) \frac{\partial^2 B}{\partial P_1^{\nu} \partial X_2^{\nu}}(\tau, z_2) \\
&+\underbrace{X_2'^{\mu}(\tau, z_1) X_2^{\nu}}_{}(\tau, z_2) \frac{\partial^2 A}{\partial X_1'^{\mu} \partial X_2'^{\mu}}(\tau, z_1) \frac{\partial^2 B}{\partial P_1^{\nu} \partial X_2^{\nu}}(\tau, z_2) \\
&+\underbrace{X_2^{\mu}(\tau, z_1) X_2'^{\nu}}_{}(\tau, z_2) \frac{\partial^2 A}{\partial X_1'^{\mu} \partial X_2^{\mu}}(\tau, z_1) \frac{\partial^2 B}{\partial P_1^{\nu} \partial X_2'^{\nu}}(\tau, z_2) \\
&+\underbrace{X_2'^{\mu}(\tau, z_1) X_2'^{\nu}}_{}(\tau, z_2) \frac{\partial^2 A}{\partial X_1'^{\mu} \partial X_2'^{\mu}}(\tau, z_1) \frac{\partial^2 B}{\partial P_1^{\nu} \partial X_2'^{\nu}}(\tau, z_2))
\end{aligned}
$$

$$
\begin{aligned}
&+\underbracket{P_2^\mu(\tau,z_1)P_2^\nu}(\tau,z_2)\frac{\partial^2 A}{\partial X_1'^\mu \partial P_2^\mu}(\tau,z_1)\frac{\partial^2 B}{\partial P_1^\nu \partial P_2^\nu}(\tau,z_2)))\\
&-(A\longleftrightarrow B)\}
\end{aligned}
\tag{B5}
$$

where subscript index A denotes antisymmetric combination of the contractions in the braces and $X'^\nu(\tau,z)\equiv iz\frac{d}{dz}X^\nu(\tau,z)$. In the case under consideration the constraints are quadratic in momenta. It means that the terms corresponding to $\hbar^n$, $n>2$ are absent in the $*-$commutator and $\Delta^{(2)}_{[A,B]}$ is the only quantum contribution.

Taking into account the ordering prescription given in Appendix A we find after straightforward calculations

$$
\begin{aligned}
&\underbracket{X^\mu(\tau,z_1)X^\nu}(\tau,z_2)\\
&= \frac{\alpha'\hbar}{2}\eta^{\mu\nu}\sum_{n>0}\frac{1}{n}[(z_1/z_2)^n+(z_2/z_1)^n]+i\hbar\tau\eta^{\mu\nu}\\
&+ \frac{\alpha'\hbar}{2}(\alpha_0+\beta_0)\frac{(1-\mu)^2}{\mu D}\eta^{\mu\nu}\sum_{n>0}\frac{1}{n}[(\mu z_1/z_2)^n+(\mu z_2/z_1)^n],\\
&\underbracket{X^\mu(\tau,z_1)P^\nu}(\tau,z_2)\\
&= \frac{1}{2}\frac{i\hbar}{2\pi}\eta^{\mu\nu}(1+\sum_{n>0}[(z_1/z_2)^n+(z_2/z_1)^n])\\
&+ \frac{1}{2}\frac{i\hbar}{2\pi}(\alpha_0-\beta_0)\frac{(1-\mu)^2}{\mu D}\eta^{\mu\nu}\sum_{n>0}\frac{1}{n}[(\mu z_1/z_2)^n+(\mu z_2/z_1)^n],\\
&\underbracket{P^\mu(\tau,z_1)P^\nu}(\tau,z_2)\\
&= \frac{1}{2}\frac{\hbar}{(2\pi)^2\alpha'}\eta^{\mu\nu}\sum_{n>0}n[(z_1/z_2)^n+(z_2/z_1)^n]\\
&+ \frac{1}{2}\frac{\hbar}{(2\pi)^2\alpha'}(\alpha_0+\beta_0)\frac{(1-\mu)^2}{\mu D}\eta^{\mu\nu}\sum_{n>0}n[(\mu z_1/z_2)^n+(\mu z_2/z_1)^n]
\end{aligned}
\tag{B6}
$$

Since $|\mu|<1$ and $|z_1|=|z_2|=1$ all power series in the variables $\mu z_1/z_2$ and $\mu z_2/z_1$ in the eqs.(B6) are convergent. However, power series in the variables z_1/z_2 and z_2/z_1 diverge.[11] Moreover, the singular point $z_1=z_2$

[11]Note that divergent series are absent in the case of normal ordering contractions (see ref. [26]).

in the divergent series is situated on the contour of integration. Hence the corresponding integrals are ill defined and a regularization is required.

To regularize the integrals and contractions we change z_1/z_2 by $(z_1/z_2)e^{-\epsilon}$ and z_2/z_1 by $(z_2/z_1)e^{-\epsilon}$ in the divergent series. Here $\epsilon > 0$ is the regularization parameter. As a result all series in the eqs.(B6) are summed to elementary functions and contractions take the form:

$$
\begin{aligned}
&\underbracket{X^{\mu}(\tau, z_1)X^{\nu}}(\tau, z_2) \\
&= \frac{\alpha'\hbar}{2}\eta^{\mu\nu}[\ln(1 - z_1e^{-\epsilon}/z_2) + \ln(1 - z_2/(e^{\epsilon}z_1))] + i\hbar\tau\eta^{\mu\nu} \\
&+ \frac{\alpha'\hbar}{2}(\alpha_0 + \beta_0)\frac{(1-\mu)^2}{\mu D}\eta^{\mu\nu}[\ln(1 - \mu z_1/z_2) + \ln(1 - \mu z_2/z_1)], \\
&\underbracket{X^{\mu}(\tau, z_1)P^{\nu}}(\tau, z_2) \\
&= \frac{1}{2}\frac{i\hbar}{2\pi}\eta^{\mu\nu}\Big(\frac{z_1e^{-\epsilon}}{z_2 - z_1e^{-\epsilon}} - \frac{z_1e^{\epsilon}}{z_2 - z_1e^{\epsilon}}\Big) \\
&+ \frac{1}{2}\frac{i\hbar}{2\pi}(\alpha_0 - \beta_0)\frac{(1-\mu)^2}{\mu D}\eta^{\mu\nu}\Big(\frac{\mu z_1}{z_2 - \mu z_1} + \frac{\mu^{-1}z_1}{z_2 - \mu^{-1}z_1}\Big) \\
&\underbracket{P^{\mu}(\tau, z_1)P^{\nu}}(\tau, z_2) \\
&= \frac{1}{2}\frac{\hbar}{(2\pi)^2\alpha'}\eta^{\mu\nu}\Big(\frac{z_2z_1e^{-\epsilon}}{(z_2 - z_1e^{-\epsilon})^2} - \frac{z_2z_1e^{\epsilon}}{(z_2 - z_1e^{\epsilon})^2}\Big) \\
&+ \frac{1}{2}\frac{\hbar(\alpha_0 + \beta_0)}{(2\pi)^2\alpha'}\frac{(1-\mu)^2}{\mu D}\eta^{\mu\nu}\Big(\frac{\mu z_2z_1}{(z_2 - \mu z_1)^2} + \frac{\mu^{-1}z_2z_1}{(z_2 - \mu^{-1}z_1)^2}\Big) \qquad \text{(B7)}
\end{aligned}
$$

Now the integrals in the eqs.(B5) can be calculated by standard methods and at the end of calculations we should set $\epsilon \to 0$.

We will describe briefly the procedure of calculation of the integrals. First, integrate over the variable z_2. As it has been noted the regularization leads to the situation when two singular points $z_2 = z_1e^{-\epsilon}$ and $z_2 = z_1e^{\epsilon}$ arising in the integrand turned out to be situated on the opposite sides of contour. The only pole in the case under consideration is $z_2 = z_1e^{-\epsilon}$. Thus, this regularization procedure can in principle be treated as a some sort of point splitting adapted for use within the canonical formalism.

The above procedure leads, for example, to the following result

$$[\underbracket{X^{\mu}(\tau, z_1)P^{\nu}}(\tau, z_2)]_A$$

$$= \quad \frac{1}{2}\frac{i\hbar}{2\pi}\eta^{\mu\nu}\Big(\frac{z_1 e^{-\epsilon}}{z_2 - z_1 e^{-\epsilon}} - \frac{z_1 e^{\epsilon}}{z_2 - z_1 e^{\epsilon}}\Big) \tag{B8}$$

Since the eq.(B8) or its derivative with respect to z_1 is contained in all $*-$commutators (see (B3)) residues in all the poles[12] except $z_2 = z_1 e^{-\epsilon}$ will vanish if $\epsilon \to 0$. The method of calculation of the integrals within this regularization procedure is consistent with the method described in ref. [33] and used then in ref. [26] where boundary of the ring $(|z_2| > |z_1|) - (|z_1| > |z_2|)$ played role of a contour for non-regularized integrals. In order to compute $*-$commutators of the constraints symbols (21) we should replace A and B in the eq.(B5) by the L_n or the $\bar{L}_n$ and then integrate the obtained expressions according to the above prescription.

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